ADVANCED SUBSIDIARY GCE

MATHEMATICS 4732

Probability & Statistics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
*   Printed Answer Book 4732
*   List of Formulae (MF1)

Other Materials Required:
*   Scientific or graphical calculator

Friday 18 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
*   **The questions are on the inserted Question Paper.**
*   **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully and make sure that you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a graphical calculator in this paper.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
*   **You are reminded of the need for clear presentation in your answers.**
*   The total number of marks for this paper is **72**.
*   The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

*   Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

1   The marks of some students in a French examination were summarised in a grouped frequency distribution and a cumulative frequency diagram was drawn, as shown below.
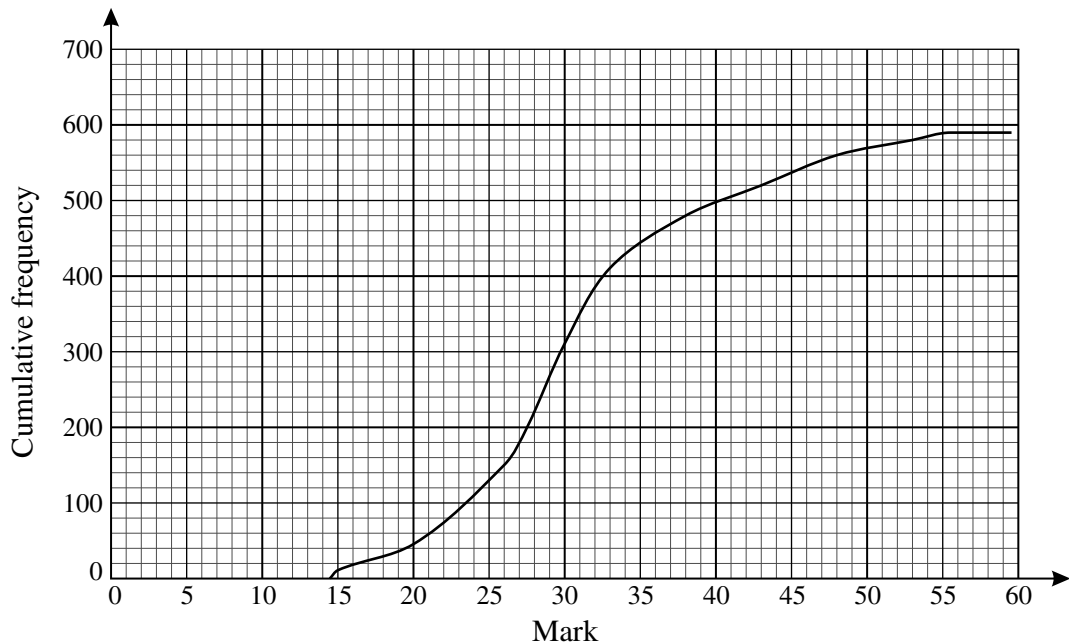
(i)   Estimate how many students took the examination.   **[1]**

(ii) How can you tell that no student scored more than 55 marks?   **[1]**

(iii) Find the greatest possible range of the marks.   **[1]**

(iv) The minimum mark for Grade C was 27. The number of students who gained exactly Grade C was the same as the number of students who gained a grade lower than C. Estimate the maximum mark for Grade C.   **[3]**

(v)   In a German examination the marks of the same students had an interquartile range of 16 marks. What does this result indicate about the performance of the students in the German examination as compared with the French examination?   **[3]**

2   Three skaters, *A*, *B* and *C*, are placed in rank order by four judges. Judge *P* ranks skater *A* in 1st place, skater *B* in 2nd place and skater *C* in 3rd place.

(i)   Without carrying out any calculation, state the value of Spearman's rank correlation coefficient for the following ranks. Give a reason for your answer.   **[1]**

Skater	*A*	*B*	*C*
Judge *P*	1	2	3
Judge *Q*	3	2	1

(ii) Calculate the value of Spearman's rank correlation coefficient for the following ranks.   **[3]**

Skater	*A*	*B*	*C*
Judge *P*	1	2	3
Judge *R*	3	1	2

(iii) Judge *S* ranks the skaters at random. Find the probability that the value of Spearman's rank correlation coefficient between the ranks of judge *P* and judge *S* is 1.   **[3]**

3   **(i)** Some values, (x, y), of a bivariate distribution are plotted on a scatter diagram and a regression line is to be drawn. Explain how to decide whether the regression line of y on x or the regression line of x on y is appropriate. **[2]**

(ii) In an experiment the temperature, $x\,°\text{C}$, of a rod was gradually increased from $0\,°\text{C}$, and the extension, $y\,\text{mm}$, was measured nine times at $50\,°\text{C}$ intervals. The results are summarised below.

$$n = 9 \quad \Sigma x = 1800 \quad \Sigma y = 14.4 \quad \Sigma x^2 = 510\,000 \quad \Sigma y^2 = 32.6416 \quad \Sigma xy = 4080$$

   (a) Show that the gradient of the regression line of y on x is 0.008 and find the equation of this line. **[4]**

   (b) Use your equation to estimate the temperature when the extension is 2.5 mm. **[1]**

   (c) Use your equation to estimate the extension for a temperature of $-50\,°\text{C}$. **[1]**

   (d) Comment on the meaning and the reliability of your estimate in part **(c)**. **[2]**

4   **(i)** The random variable W has the distribution $\text{B}\left(10, \frac{1}{3}\right)$. Find

   (a) $\text{P}(W \leqslant 2)$, **[1]**

   (b) $\text{P}(W = 2)$. **[2]**

(ii) The random variable X has the distribution $\text{B}(15, 0.22)$.

   (a) Find $\text{P}(X = 4)$. **[2]**

   (b) Find $\text{E}(X)$ and $\text{Var}(X)$. **[3]**

5   Each of four cards has a number printed on it as shown.

$$\boxed{1} \qquad \boxed{2} \qquad \boxed{3} \qquad \boxed{3}$$
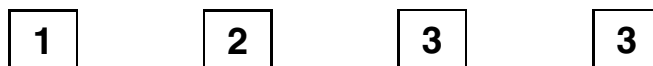
Two of the cards are chosen at random, without replacement. The random variable X denotes the sum of the numbers on these two cards.

   (i) Show that $\text{P}(X = 6) = \frac{1}{6}$ and $\text{P}(X = 4) = \frac{1}{3}$. **[3]**

   (ii) Write down all the possible values of X and find the probability distribution of X. **[4]**

   (iii) Find $\text{E}(X)$ and $\text{Var}(X)$. **[5]**

6   There are 10 numbers in a list. The first 9 numbers have mean 6 and variance 2. The 10th number is 3. Find the mean and variance of all 10 numbers. **[6]**

[Questions 7 and 8 are printed overleaf.]

7 The menu below shows all the dishes available at a certain restaurant.

Rice dishes	Main dishes	Vegetable dishes
Boiled rice	Chicken	Mushrooms
Fried rice	Beef	Cauliflower
Pilau rice	Lamb	Spinach
Keema rice	Mixed grill	Lentils
	Prawn	Potatoes
	Vegetarian	

A group of friends decide that they will share a total of 2 different rice dishes, 3 different main dishes and 4 different vegetable dishes from this menu. Given these restrictions,

  (i) find the number of possible combinations of dishes that they can choose to share, **[3]**

 (ii) assuming that all choices are equally likely, find the probability that they choose boiled rice.

[2]

The friends decide to add a further restriction as follows. If they choose boiled rice, they will not choose potatoes.

(iii) Find the number of possible combinations of dishes that they can now choose. **[3]**

8 The proportion of people who watch *West Street* on television is 30%. A market researcher interviews people at random in order to contact viewers of *West Street*. Each day she has to contact a certain number of viewers of *West Street*.

  (i) Near the end of one day she finds that she needs to contact just one more viewer of *West Street*. Find the probability that the number of further interviews required is

    (a) 4, **[3]**

    (b) less than 4. **[3]**

 (ii) Near the end of another day she finds that she needs to contact just two more viewers of *West Street*. Find the probability that the number of further interviews required is

    (a) 5, **[4]**

    (b) more than 5. **[2]**

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geq$ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1i	590	B1	1	Allow approximately 590
ii	Graph horiz (for $\geq$ 55 mks) oe	B1	1	or levels off, or grad = 0, grad not increase Allow line not rise, goes flat, plateaus, stops increasing, not increase, doesn't move
iii	39 to 41	B1	1	
iv	Attempt read cf at 26 or 27 Double & attempt read x Max C = 29 to 31.5	M1 M1 A1	 3	eg 26 mks $\rightarrow$ 150th 27 mks $\rightarrow$ 180th eg read at cf = 300 or 360 Indep of first M1 May be implied by ans Answer within range, no working, M1M1A1 32 without working, sc B1
v	LQ = 25.5-26.5 or UQ = 34-35.5 IQR = 8-10 (German) more spread	M1 A1 B1ft	 3	M1 for one correct quartile dep $\geq$ 1 correct quartile or no working or less consistent, less uniform, less similar, more varied, more variable, greater variance, more spaced apart, further apart ft their IQR; must be consistent with IQR Correct comment with no working: M0A0B1
Total		**9**		
2i	Opposite orders or ranks or scores or results or marks $r_s = -1$	 B1	 1	or reversed, or backwards, or inverse or as one increases the other decreases Needs reason AND value
ii	Attempt Σd^2 (= 6) $1 - \frac{6 \times \Sigma d^2}{3(3^2 - 1)}$ $= -\frac{1}{2}$ oe	M1 M1 A1	 3	 dep 1st M1 Allow use wrong table for M1M1
iii	3! or $^3P_3$ or 6 1 ÷ their '6' $\frac{1}{6}$ oe eg $\frac{6}{36}$	M1 M1 A1	 3	r attempt list possible orders of 1,2,3 ($\geq$3 orders) 2nd M1 for fully correct method only or $\frac{1}{3} \times \frac{1}{2} (\times 1)$: M1M1
Total		**7**		
3i	If x is contr (or indep) or y depend't, use y on x If neither variable contr'd (or indep) AND want est y from x: use y on x	 B1 B1	 2	Allow x increases constantly, is predetermined, you choose x, you set x, x is fixed, x is chosen Allow y not controlled AND want est y from x Ignore incorrect comments
iia	$S_{xx} = 510000 - \frac{1800^2}{9}$ (= 150000) $S_{xy} = 4080 - \frac{1800 \times 14.4}{9}$ (= 1200) $b = \frac{'1200'}{'150000'}$ (= 0.008) $y - \frac{14.4}{9} = 0.008(x - \frac{1800}{9})$ $y = 0.008x$ (+ 0)	 M1 M1 M1 A1	 4	or $\frac{510000}{9} - 200^2$ (= 16666.7) or $\frac{4080}{9}$ - 200×1.6 (= 133.33) M1 for either S $b = \frac{'133.33'}{'16666.7'}$ dep correct expressions both S's or $a = \frac{14.4}{9} - 0.008 \times \frac{1800}{9}$ (= 0) Must be all correct for M1 CAO
iib	312.5 or 313	B1ft	1	ft their equn in (iia)
iic	−0.4	B1ft	1	ft their equn in (iia)

iid	Contraction oe	B1(ft)	or length decreased, shorter, pushed in, shrunk, smaller
	Unreliable because extrapolated oe	B1 2	or not in the range of x or not in range of previous results
Total		**10**	
4ia	0.299 (3 sf)	B1 1	
ib	0.2991 – 0.1040	M1	Must subtract correct pair from table
	= 0.195 (3 sf) or $\frac{1280}{6561}$ oe	A1 2	
iia	$^{15}C_4 \times (1-0.22)^{11} \times 0.22^4$	M1	Allow M1 for $^{15}C_4 \times 0.88^{11} \times 0.22^4$
	= 0.208 (3 sf)	A1 2	
iib	$(15 \times 0.22 =)\ 3.3$	B1	
	$15 \times 0.22 \times (1-0.22)$ or '3.3'$\times(1-0.22)$	M1	Allow M1 for $15 \times 0.22 \times 0.88$
	= 2.57 (3 sf)	A1 3	
Total		**8**	
5i	$\frac{1}{2} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{1}{3}$ or $\frac{1}{^4C_2}$ or $\frac{2}{12}$ $(= \frac{1}{6}\ \mathbf{AG})$	B1	or 1 out of 6 or 2 out of 12 or $\frac{2!}{4!} \times 2$
	$\frac{1}{4} \times \frac{2}{3}$ or $2 \times \frac{1}{4} \times \frac{1}{3}$ or $\frac{1}{2} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{1}{3}$	B1	or $\frac{2}{12}$ or $\frac{1}{6}$ or $\frac{1}{3!}$ or $\frac{1}{^4C_2}$ or $\frac{2!}{4!} \times 2$
	Add two of these or double one $(= \frac{1}{3}\ \mathbf{AG})$	B1 3	or $\frac{2}{^4C_2}$ or $4 \times \frac{1}{4} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{2}{3}$ or $\frac{4}{12}$ or $\frac{2!}{4!} \times 4$ B1B1 or $\frac{2}{6}$ or $2 \times \frac{1}{6}$ or $\frac{2}{3!}$ or $\frac{2!}{3!}$ B1B1
ii	$X = 3, 4, 5, 6$ only, stated or used	B1	Allow repetitions Allow other values with zero probabilities.
	P(X =5) wking as for P(X =4) above or $1 - ($ " $\frac{1}{6}$ " $+ \frac{1}{3} + \frac{1}{6})$ or $\frac{1}{3}$	M1	
	P(X =3) wking as for P(X =6) above or $1 - (\frac{1}{3} + $ " $\frac{1}{3}$ " $+ \frac{1}{6})$ or $\frac{1}{6}$	M1	or M1 for total of their probs = 1, dep B1
	$\begin{array}{cccc} 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$ oe	A1 4	or P(X =3)= $\frac{1}{6}$, P(X = 4)= $\frac{1}{3}$, P(X = 5)= $\frac{1}{3}$, P(X = 6)= $\frac{1}{6}$ Complete list of values linked to probs
iii	Σxp	M1	$\geq$ 2 terms correct ft
	$= 4\frac{1}{2}$	A1	
	$\Sigma x^2 p$ $(= 21\frac{1}{6})$	M1	$\geq$ 2 terms correct ft
	$- ' 4\frac{1}{2} '^2$	M1	Independent except dependent on +ve result
	$= \frac{11}{12}$ or 0.917 (3 sf)	A1 5	
Total		**12**	

4

6	$m = (9 \times 6 + 3) \div 10$	M1	or ((Sum of any 9 nos totalling 54) + 3) ÷ 10
	$= 5.7$	A1	
	$2 = \dfrac{\Sigma x^2}{9} - 6^2$	M1	or $\dfrac{\Sigma(x-6)^2}{9} = 2$ M1
	$\Sigma x^2 = 2 \times 9 + 6^2 \times 9$ or 342	A1	or $\Sigma x^2 = 18 + 12 \times 54 - 36 \times 9$ or 342 A1
	$v = \dfrac{('342' + 3^2)}{10} - '5.7'^2$	M1	dep Σx^2 attempted, eg $(\Sigma x)^2 (= 3249)$ or just state 'Σx^2'; allow √
	$= 2.61$ oe	A1 6	CAO
Total		**6**	
7i	${}^4C_2 \times {}^6C_3 \times {}^5C_4$ or 6×20×5	M1M1	M1 for any 2 correct combs seen, even if added
	$= 600$	A1 3	
ii	$\dfrac{2}{4}$ or $\dfrac{{}^3C_1}{{}^4C_2}$ or $\dfrac{{}^3C_1 \times {}^6C_3 \times {}^5C_4}{{}^4C_2 \times {}^6C_3 \times {}^5C_4}$ or	M1	or $\frac{1}{4} \times 1 + \frac{3}{4} \times \frac{1}{3}$ or $\frac{1}{4} \times 2$ or $\frac{1}{4} + \frac{1}{4}$
	$\dfrac{{}^3C_1 \times {}^6C_3 \times {}^5C_4}{'600'}$		
	$= \frac{1}{2}$ oe	A1 2	
iii	${}^3C_1 \times {}^6C_3 \left(\times {}^4C_4\right) + {}^3C_2 \times {}^6C_3 \times {}^5C_4$	M1M1	M1 either product seen, even if × or ÷ by something
	360	A1 3	
Total		**8**	

8			
8ia	Geo(0.3) stated or implied $0.7^3 \times 0.3$ = 0.103 (3 sf)	M1 M1 A1 3	by $0.7^n \times 0.3$
b	0.7^3 or 0.343 $1 - 0.7^3$ = 0.657	M1 M1 A1 3	0.7^3 must be alone, ie not $0.7^3 \times 0.3$ or similar allow $1 - 0.7^4$ or 0.7599 or 0.76 for M1 only or $0.3 + 0.7 \times 0.3 + 0.7^2 \times 0.3$: M1M1 1 term wrong or omitted or extra M1 or $1 - (0.3 + 0.7 \times 0.3 + 0.7^2 \times 0.3)$ or 0.343: M1
iia	State or imply one viewer in 1st four $^4C_1 \times 0.7^3 \times 0.3$ (= 0.412) $\times 0.3$ = 0.123 (3 sf)	M1 M1 M1 A1 4	or B(4, 0.3) stated, or $^4C_1$ used, or YNNNY dep 1st M1
b	$0.7^5 + {}^5C_1 \times 0.7^4 \times 0.3$ = 0.528 (3 sf)	M1 A1 2	or $1 - (0.3^2 + 2 \times 0.3^2 \times 0.7 + 3 \times 0.3^2 \times 0.7^2 + 4 \times 0.3^2 \times 0.7)$ Not ISW, eg $1 - 0.528$: M1A0
Total		**12**	

Total 72 marks